SAFE HANDS & IIT-ian's PACE

LEAP TEST# 06 (JEE) ANS KEY Dt. 14-12-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	C	31	A	61	A
2	C	32	D	62	C
3	C	33	A	63	C
4	A	34	B	64	D
5	B	35	A	65	B
6	B	36	B	66	C
7	A	37	B	67	D
8	C	38	C	68	B
9	B	39	D	69	C
10	C	40	D	70	A
11	B	41	A	71	B
12	D	42	C	72	A
13	B	43	D	73	BONUS
14	D	44	B	74	D
15	B	45	D	75	A
16	A	46	C	76	B
17	D	47	A	77	B
18	D	48	A	78	D
19	C	49	C	79	C
20	A	50	A	80	D
21	5.16	51	3	81	12
22	1.4	52	11.5	82	9
23	0.67	53	4	83	9
24	3	54	4	84	-8
25	3	55	3	85	96

LT-06 (JEE) PHYSICS SOLUTION

: HINTS AND SOLUTIONS :

Single Correct Answer Type

1 **(c)**

$$\frac{40}{80}=\frac{R^2}{(R+h)^2}=\sqrt{2}R=R+h$$

$h=(\sqrt{2}-1)R$

2 **(c)**

As we decrease the magnitude of mechanical energy of the spacecraft-earth system, it means we are increasing the energy of the spacecraft-earth system as the total energy of the bounded system is negative

As we change the energy, the circular orbit of the spacecraft will become elliptical. Let a be the semi-major axis of this new elliptical orbit

$$E_{\text{final}}=-\frac{GM_m}{2a}$$

$E_{\text{initial}}=\frac{GM_m}{2r}$, where $r=7000$ km

$$=\frac{GM_m}{2r}=-\frac{GM_m}{2a}$$

$$a=\frac{r}{0.9}=1.11\,r$$

$r_{\max}=2a$ [where $r_{\max}$ is the distance corresponding to aphelion] $=2.22r-r=1.22\,r$

Required greatest height, $h=r_{\max}-R_e=2140$ km

3 **(c)**

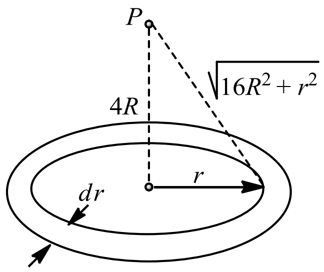
$$F_G=\frac{GM^2}{4R^2}\Rightarrow\frac{mv^2}{R}=\frac{GM^2}{4R^2}$$

$$v=\frac{1}{2}\sqrt{\frac{GM}{R}}$$

4 **(a)**

$$W=\Delta U=U_f-U_i=U_\infty-U_P$$
$$=-U_P=-mV_P$$
$$=-V_P(\text{as } m=1)$$

Potential at point P will be obtained by in integration as given below. Let dM be the mass of small rings as shown



$$dM=\frac{M}{\pi(4R)^2-\pi(3R)^2}(2\pi r)dr$$

$$=\frac{2Mr\,dr}{7R^2}$$

$$dV_P=-\frac{G.dM}{\sqrt{16R^2+r^2}}$$

$$=-\frac{2GM}{7R^2}\int_{3R}^{4R}\frac{r}{\sqrt{16R^2+r^2}}\cdot dr$$

$$=-\frac{2GM}{7R}\left(4\sqrt{2}-5\right)$$

$$\therefore\quad W=+\frac{2GM}{7R}(4\sqrt{2}-5)$$

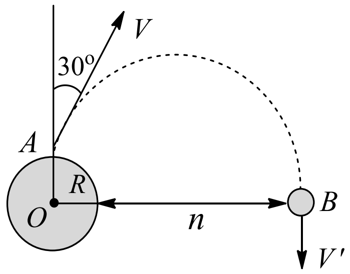
5 **(b)**

$F_G=\frac{GMm}{R}$ (as $F_G\propto\frac{1}{R}$ given)

So $\frac{mv^2}{R}=\frac{GMm}{R}\Rightarrow v^2\propto R^0$

6 **(b)**

Conservation of angular momentum of the body about O yield the following :



$(mv\sin 30°)\ R=nV'(R+h)$

$$\frac{V}{2}R=\frac{V}{4}(R+h)\quad\left[\therefore V'=\frac{V}{4}\right]$$

Therefore, $h=R$

7 **(a)**

Let us first calculate the mass of the inner solid sphere of radius r

Mass of the inner solid sphere is

$$M'=\frac{M_e}{\frac{4}{3}\pi R_e^3}\times\frac{4}{3}=\pi r^2=\frac{M_e}{R_e^3r^3}$$

Now, $g=\frac{GM_er^3}{R_e^3}\times\frac{1}{r^2}$ or $g=\frac{GM_er}{R_e^3}$

Force on the particle of mass $m=mg$

$$=\frac{GM_emr}{R_e^3}$$

Note: ***Those layers of the sphere which have radii larger than r do not have contribution to the gravitational force***

8 **(c)**

If x_1 and x_2 are the distances covered by the two bodies, then $x_1+x_2=9R$

Also, $Mx_1=5Mx_2\ \Rightarrow x_2=\frac{x_1}{5}$

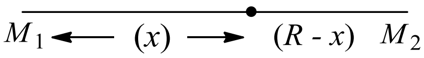
$x_1 + \frac{x_1}{5} = 9R \;\Rightarrow\; x_1 = 7.5\,R$

9 **(b)**

$g' = g\left[1 - \frac{2h}{R}\right] = g\left[1 - \frac{d}{R}\right]$

$\frac{2h}{R} = \frac{d}{R} \Rightarrow d = 2h$

10 **(c)**



$\frac{GM_1}{x^2} = \frac{GM_2}{(R-x)^2}$

$\frac{M_2}{M_1}x^2 = R^2 + x^2 - 2Rx$

Let $\frac{M_2}{M} = k$

$x^2(k-1) + 2Rx - R^2 = 0$

$x = -\frac{2R + \sqrt{4R^2 + 4(k-1)R^2}}{2(k-1)}$

$= \frac{R\sqrt{M_1}}{\sqrt{M_1} + \sqrt{M_2}}$

$R - x = \frac{R\sqrt{M_2}}{\sqrt{M_1} + \sqrt{M_2}}$

Gravitational potential at point P is

$-\left(\frac{GM_1}{x} + \frac{GM_2}{R-x}\right)$

$= -\left[\frac{GM_1(\sqrt{M_1} + \sqrt{M_2})}{R\sqrt{M_1}} + \frac{GM_2(\sqrt{M_1} + \sqrt{M_2})}{R\sqrt{M_2}}\right]$

$= -\left[\frac{G(\sqrt{M_2} + \sqrt{M_1})}{R}(\sqrt{M_1} + \sqrt{M_2})\right]$

$= -\frac{G(\sqrt{M_1} + \sqrt{M_2})^2}{R}$

11 **(b)**

$U = \frac{a}{x^2} - \frac{b}{x}; F = -\frac{dU}{dx} = -\left[\frac{-2a}{x^3} . \frac{b}{x^2}\right]$

At equilibrium position, $F = 0$

$x = x_0 = \frac{2a}{b}$

At $x = x_0 + \Delta$, i.e., at a displacement of $\Delta x (\ll x_0)$ from mean position,

$F = -\left[\frac{bx - 2a}{x^3}\right] = -\left[\frac{b(x_0 + \Delta x) - 2a}{(x_0 + \Delta x)^3}\right] = \frac{b\Delta x}{x_0^3}$

$= -\frac{b^4}{8a^3}\Delta x$

As $F\ \mu \Delta x$, so particle performs simple harmonic motion with time period

$T = 2\pi\sqrt{\frac{8a^3 m}{b^4}}$

12 **(d)**

$T = 8\text{s}, \omega = \frac{2\pi}{T} = \left(\frac{\pi}{4}\right)\text{rads}^{-1}$

$x = A\sin\omega t$

$\therefore \quad a = -\omega^2 x = -\left(\frac{\pi^2}{16}\right)\sin\left(\frac{\pi}{4}t\right)$

Substituting $t = \frac{4}{3}$ s, we get

$a = -\left(\frac{\sqrt{3}}{32}\pi^2\right)\text{cms}^{-2}$

13 **(b)**

$y_1 = a\sin\omega t$ and $y_2 = \sin(\omega t + \phi)$

$y_2 - y_1 = a\sqrt{2} = a\sin(\omega t + \phi) - a\sin\omega t)$

or $\sqrt{2}a = 2a\cos\left(\frac{\omega t + \phi + \omega t}{2}\right) = \sin\left(\frac{\omega t + \phi - \omega t}{2}\right)$

$= 2a\cos\left(\omega t + \frac{\phi}{2}\right)\sin\frac{\phi}{2}$

For maximum value, $\cos(\omega t + \phi) = 1$, therefore

$2\sin\frac{\phi}{2} = \sqrt{2} \Rightarrow \sin\frac{\phi}{2} = \frac{1}{\sqrt{2}}$

or $\frac{\phi}{2} = \frac{\pi}{4}$ or $\phi = \frac{\pi}{2}$

14 **(d)**

Equation are $x_1 = a\cos\left(\omega t + \frac{\pi}{6}\right)$

and $x_2 = a\cos\left(\omega t + \frac{\pi}{3}\right)$

The first will fast through the mean position when $x_1 = 0$

i.e., for instants t for which $\left(\omega t + \frac{\pi}{6}\right) = \frac{n\pi}{2}$, where n is an
integer

The smallest value of t is $n = 1, \omega t_1 = (\pi/2) = (\pi/6) = \pi/3$

The second will pass through the mean position when $x_2 = 0$,i.e., for instants t for which $\left(\omega t + \frac{\pi}{3}\right) = \frac{m\pi}{2}$

where m is an integer

The smallest value of t is $m = 1, = (\pi/2) - (\pi/3) = \pi/6$

The smallest interval between the instants $x_1 = 0$ and $x_2 = 0$ is therefore

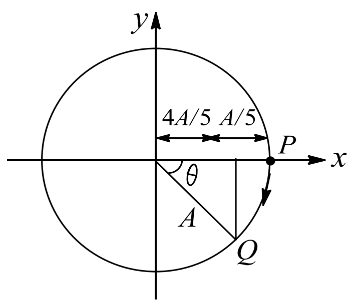
$\omega(t_1 : t_2) = \left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \frac{\pi}{6} \Rightarrow t_1 : t_2 = \frac{\pi}{6\omega}$

15 **(b)**

Method 1:

Particle is starting form rest i.e., from one of its extreme position.

As particle moves a distance $A/5$, we can represent it on a circle as shown



$$\cos\theta = \frac{4A/5}{A} = \frac{4}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\omega t = \cos^{-1}\left(\frac{4}{5}\right) \Rightarrow t = \frac{1}{\omega}\cos^{-1}\left(\frac{4}{5}\right) = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

Method 2: As the particle starts from rest, i.e., from extreme position $x = A\sin(\omega t - \phi)$

At $t = 0; x = A \Rightarrow \phi = \frac{\pi}{2}$

$$A - \frac{A}{5} = A\cos\omega t$$

$$\frac{4}{5} = \cos\omega t \Rightarrow \omega t = \cos^{-1}\frac{4}{5}$$

$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{4}{5}\right)$$

16 **(a)**

$U = k|x|^3 \Rightarrow F = -\frac{dU}{dx} = -3k|x|^2$ ...(i)

Also, for SHM $x = a\sin\omega t$ and $\frac{d^2x}{dt^2} + \omega^2 x = 0$

$\Rightarrow$ acceleration $a = \frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow F = ma$

$= m\frac{d^2x}{dt^2} = -m\omega^2 x$ ...(ii)

From equation (i) & (ii) we get $\omega = \sqrt{\frac{3kx}{m}}$

$$\Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{3kx}} = 2\pi\sqrt{\frac{m}{3k(a\sin\omega t)}} \Rightarrow T \propto \frac{1}{\sqrt{a}}$$

17 **(d)**

Since acceleration of the particle at initial moment is maximum possible and is negative; therefore, the particle is at right extreme position at this moment. When the particle is released, it starts to move to the left. It means, velocity starts to increase from zero initial value to negative value and its magnitude becomes maximum possible at mean position (at $t = T/4$). It means at $t = T/4$, kinetic energy is equal to maximum possible.

At $t = T/2$, the particle comes to instantaneous rest at left extreme position. It means at $t = T/2$, v is equal to zero. Hence kinetic energy is equal to zero. At $t = 3T/4$, particle comes back to mean position and now moves to the right. Therefore, velocity is positive and has maximum possible magnitude. Therefore. Kinetic energy is maximum possible.

At $t = T$, particle comes back to initial position (extreme right position). Velocity and kinetic energy become equal to zero

18 **(d)**

Maximum velocity $= a\omega = a\sqrt{\frac{K}{m}}$

Given that $a_1\sqrt{\frac{K_1}{m}} = a_2\sqrt{\frac{K_2}{m}} \Rightarrow \frac{a_1}{a_2} = \sqrt{\frac{K_2}{K_1}}$

Matrix Match Type

19 **(c)**

Gravitational potential is defined as the work done in carrying a unit mass from a point outside the field (zero potential) to a point inside the field. So when reference point is altered the potential will vary and the potential difference will remain the same. Also the energy is not dependent on the angle by any means and depends only on the two points (initial and final) in any conservation field like gravitational field

So i→c, d

Escape velocity is given by $\sqrt{GM/R}$ and is dependent on the mass of the planet and does not depend on the angle of projection

So ii → d

Acceleration due to gravity is given by $g = GM/R^2 = 4/3\pi R_\rho G$ and so its ratio in two planets depends on their densities when their mass is not the same

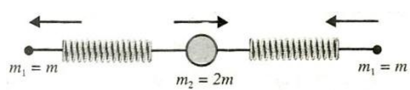
So iii → a

As the Kepler's law, the angular momentum will remain conserved for keeping the area swept in equal to remain the same. The same concept was proposed by Bohr's model of atom

So iv → b, c

20 **(a)**

1. Motion in simple harmonic



$$\frac{d^2x}{dt^2} = \frac{-k}{m_1 m_2}(m_2 + 2m_1)x$$

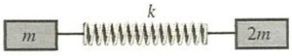
$$\omega^2 = k\left(\frac{1}{m_1} + \frac{2}{m_2}\right) = k\left(\frac{1}{m} + \frac{2}{2m}\right)x$$

$$\omega = \sqrt{\frac{2k}{m}}$$

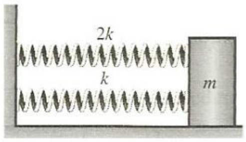
$\therefore$ Frequency $= \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

ii. Angular frequency $\omega = \sqrt{\frac{k(m+2m)}{2m^2}} = \sqrt{\frac{3k}{2m}}$

$\therefore$ Frequency $= \frac{1}{2\pi}\sqrt{\frac{3k}{2m}}$



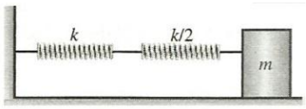
iii. Here effective spring constant $= 3k$



$\therefore$ Frequency $= \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

iv. effective spring constant (K) is given by

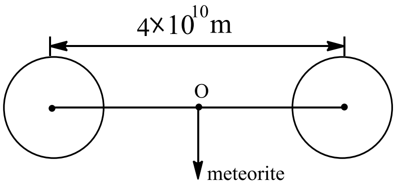
$$\frac{1}{k} = \frac{1}{k} + \frac{1}{\frac{k}{2}} = \frac{3}{k}$$



$k = \frac{k}{3}$ $\quad\therefore$ Frequency $= \frac{1}{2\pi}\sqrt{\frac{k}{3m}}$

Integer Answer Type

21 **(5.16)**



Using energy conservation principle at points O and ∞,

$$(\mathrm{TE})_{\mathrm{at\ O}} = (\mathrm{TE})_{\mathrm{at\ \infty}}$$

$$\Rightarrow \frac{-GMm}{r} - \frac{GMm}{r} + \frac{1}{2}mv^2 = 0 + 0$$

$$\Rightarrow v = \sqrt{\frac{4GM}{r}}$$

$$= \sqrt{\frac{4 \times 6.67 \times 10^{-11} \times 2 \times 10^{31}}{2 \times 10^{10}}}$$

$$= \frac{2 \times 10^{10}}{10^5}\sqrt{6.67}$$

$$= 2 \times 2.58 \times 10^5$$

$$= 5.16 \times 10^5 \ \mathrm{m/s}$$

22 **(1.4)**

As $y = pt^2$

$$\therefore \frac{dy}{dt} = pt$$

$$\Rightarrow \frac{d^2y}{dt^2} = 2p = 2 \times (2) = 4\mathrm{ms}^{-2}$$

As $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow T^2 = 4\pi^2\frac{L}{g}$

$$\Rightarrow \frac{T_1^2}{T_2^2} = \frac{g_2}{g_1} = \frac{g+a}{g} = \frac{14}{10} = \frac{7}{5} = 1.4$$

23 **(0.67)**

Let θ_1 and θ_2 be angular displacement of first and second pendulum respectively at an instant. θ_0 be the maximum angular displacement of each pendulum. Let ϕ_1 and ϕ_2 be the initial phases of two pendulums.

Then, $\theta_1 = \theta_0 \sin(\omega t + \phi_1)$ ... (i)

$\theta_2 = \theta_0 \sin(\omega t + \phi_2)$ ... (ii)

for 1st pendulum,

$0_1 = 4^0 - \theta_0$

From (i),

$4 = 4\sin(\omega t + \phi_1)$

$\Rightarrow \sin(\omega t + \phi_1) = 1$

$\Rightarrow \omega t + \phi_1 = 90^0$ .. (iii)

For 2nd pendulum,

$\theta_2 = -2$

From (ii),

$-2 = 4\sin(\omega t + \phi_2)$

$\Rightarrow \sin(\omega t + \phi_2) = -\frac{1}{2}$

$\Rightarrow (\omega t + \phi_2) = 210^0$ (iv)

Subtracting (iii) from (iv), we get

$$(\omega t + \phi_2) - (\omega t + \phi_1) = 210^0 - 90^0$$

$$= 120^0$$

$$= \frac{2}{3}\pi = 0.67\pi$$

24 **(3)**

$y = 4\cos^2\left(\frac{t}{2}\right)\sin 1000\,t$

$\Rightarrow y = 2(1 + \cos t)\sin 1000\,t$

$\Rightarrow y = 2\sin 1000\,t + 2\cos t \sin 1000\,t$

$\Rightarrow y = 2\sin 1000\,t + \sin 999\,t + \sin 1001\,t$

It is a sum of three S.H.M.

25 **(3)**

$y = 8\sin^2\left(\frac{t}{2}\right)\sin(10t)$

$= 4[1 - \cos t]\sin(10\,t)$(using $2\sin^2\frac{\theta}{2} = 1 - \cos\theta$)

$= 4\sin(10t) - 4\sin(10t)\cos t$

$= 4\sin(10t) - 2[\sin 11t + \sin 9t]$

(using $2\ \sin C\cos D = \sin(C + D) + \sin(C - D)$)

$= 4\sin(10\,t) - 2\sin(11t) - 2\sin(9t)$

Evidently, y is obtained as the superimposition of three independent (i.e., having different angular frequency ω) SHMs

LT 06 (JEE) Maths Solutons

: ANSWER KEY :

61)	a	62)	c	63)	c	64)	d	81)	12	82)	9	83)	9
65)	b	66)	c	67)	d	68)	b	84)	-8				
69)	c	70)	a	71)	b	72)	a	85)	96				
73)	c	74)	d	75)	a	76)	b						
77)	b	78)	d	79)	c	80)	d						

LT 06 (JEE) Maths Solutions

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(a)**

If a, b, c are in GP, then $b^2 = ac$

Taking log on both sides, we get

$2\log_e b = \log_e a + \log_e c$

$\Rightarrow 2n\log_e b = n\log_e a + n\log_e c$

$\Rightarrow 2\log_e b^n = \log_e a^n + \log_e c^n$

$\Rightarrow \log_e a^n, \log_e b^n, \log_e c^n$ be in AP.

62 **(c)**

$i^2 + i^4 + i^6 + \cdots$upto $(2k+1)$ terms

$$= \frac{i^2[1-(i^2)^{2k+1}]}{1-i^2} = \frac{-1[1-(-1)^{2k+1}]}{1+1}$$

$$= \frac{-[1-(-1)]}{2} = -1$$

63 **(c)**

$x, 1, z$ are in AP, then $2 = x + z$

...(i)

And $x, 2, z$ are in GP, then $4 = xz$

...(ii)

Divide Eq.(ii) by Eq.(i), we get

$\frac{xz}{x+z} = \frac{4}{2} \Rightarrow \frac{2xz}{x+z}$=4

Hence, $x, 4, z$ will be in HP.

64 **(d)**

Let $S = \frac{1}{3\times7} + \frac{1}{7\times11} + \frac{1}{11\times15} + \ldots\infty$

$$= \frac{1}{4}\left[\left\{\frac{1}{3} - \frac{1}{7}\right\} + \left\{\frac{1}{7} - \frac{1}{11}\right\} + \ldots\right]$$

$$= \frac{1}{4}\left[\frac{1}{3} + 0\right] = \frac{1}{12}$$

65 **(b)**

Since, $\frac{x+y}{2\sqrt{xy}} = \frac{p}{q} \Rightarrow \frac{(x+y)^2}{4xy} = \frac{p^2}{q^2}$ ...(i)

On subtracting both sides by1, we get

$$\frac{(x-y)^2}{4xy} = \frac{p^2-q^2}{q^2} \ldots (ii)$$

From Eqs. (i) and (ii), we get

$$\left(\frac{x+y}{x-y}\right)^2 = \frac{p^2}{p^2-q^2} \Rightarrow \frac{x+y}{x-y} = \frac{p}{\sqrt{p^2-q^2}}$$

$$\Rightarrow \frac{2x}{2y} = \frac{p+\sqrt{p^2-q^2}}{p-\sqrt{p^2-q^2}}$$

[by componendo-dividendo rule]

$\therefore x : y = (p + \sqrt{p^2 - q^2}) : (p - \sqrt{p^2 - q^2})$

66 **(c)**

Given that, $T_m = a + (m-1)d = \frac{1}{n}$ ...(i)

And $T_n = a + (n-1)d = \frac{1}{m}$ ...(ii)

Where a and b are the first term and common difference respectively.

On solving Eqs. (i) and (ii), we get

$$a = \frac{1}{mn} \text{ and } d = \frac{1}{mn}$$

$\therefore T_{mn} = a + (mn-1)d$

$$= \frac{1}{m} + (mn-1)\frac{1}{mn} = 1$$

67 **(d)**

We have,

$\log_{x+2}(x^3 - 3x^2 - 6x + 8) = 3$

$\Rightarrow x^3 - 3x^2 - 6x + 8 = (x+2)^3$

$\Rightarrow x^3 - 3x^2 - 6x + 8 = x^3 + 6x^2 + 12x + 8$

$\Rightarrow 9x^2 + 18x = 0 \Rightarrow x = 0, -2$

$\log_{x+2}(x^3 - 3x^2 - 6x + 8)$ is defined for

$x^3 - 3x^2 - 6x + 8 > 0$ and $x + 2 > 0$

$\therefore x = 0$

68 **(b)**

Let $S = \frac{1}{1.2} + \frac{1.3}{1.2.3.4} + \frac{1.3.5}{1.2.3.4.5.6} + \ldots$

$$\therefore T_n = \frac{1.3.5.\ldots.(2n-1)}{1.2.3.\ldots.(2n-1)2n} \times \frac{(2.4.8.\ldots.2n)}{(2.4.8.\ldots.2n)}$$

$$= \frac{(2n)!}{(2n)!.\,2^n(n!)} = \frac{1}{2^n(n!)}$$

$$\therefore S = \sum T_n = \frac{1}{2.1!} + \frac{1}{2^2.\,2!} + \frac{1}{2^3.\,3!} + \ldots$$

$= e^{1/2} - 1$

69 **(c)**

Let $S = 1 + \frac{1}{2!} + \frac{1.3}{4!} + \frac{1.3.5}{6!} + \ldots\infty$

$$\therefore T_n = \frac{1.3.5.\ldots.(2n-1)}{(2n)!} \times \frac{2.4.\ldots.2n}{2.4.\ldots.2n}$$

$$= \frac{(2n)!}{(2n)!\,2^n(n)!} = \frac{1}{2^n(n)!}$$

$$\therefore S = 1 + \sum T_n = 1 + \frac{1}{2(1)!} + \frac{1}{2^2(2)!} + \ldots\infty$$

$= e^{1/2} = \sqrt{e}$

70 **(a)**

Since, $l = A + (n-1)d$

$\therefore c = a + (n-1)(b-a)$

$\Rightarrow (n-1)=\frac{c-a}{b-a}$

$\Rightarrow n=\frac{b+c-2a}{b-a}$

71 **(b)**

Since a,b,c are in H.P. Therefore,

$b=\frac{2ac}{a+c}$

$\therefore \frac{1}{\frac{2ac}{a+c}-a}+\frac{1}{\frac{2ac}{a+c}-c}$

$=\frac{a+c}{ac-a^2}+\frac{a+c}{ac-c^2}$

$=\frac{a+c}{a(c-a)}+\frac{a+c}{c(a-c)}=\frac{(a+c)(c-a)}{ac(c-a)}=\frac{1}{a}+\frac{1}{c}$

72 **(a)**

Given, $x=1+\frac{2}{1!}+\frac{2^2}{2!}+\frac{2^3}{3!}+\ldots$

$\Rightarrow x=e^2$

$\Rightarrow x^{-1}=e^{-2}$

73 **(c)**

Let S_n denote the sum of n terms. Then,

$S_n=3n^2+5$

Now,

$a_n=S_n-S_{n-1}$

$\Rightarrow a_n=(3n^2+5)-(3(n-1)^2+5)=6n-3$

$\therefore a_n=159\Rightarrow 6n-3=159\Rightarrow 6n=162\Rightarrow n=27$

74 **(d)**

We observe that the successive differences of the terms of the sequence 12,28,50,78, … are in A.P. So, let its n^{th} term be

$t_n=an^2+bn+c,$

Putting $n=1,2,3$, we get

$t_1=a+b+c\Rightarrow a+b+c=12$

$t_2=4a+2b+c\Rightarrow 4a+2b+c=28$

$t_3=9a+3b+c\Rightarrow 9a+3b+c=50$

Solving these equations, we get

$a=3, b=7, c=2$

$\therefore t_n=3n^2+7n+2$

Hence,

$\frac{12}{2!}+\frac{28}{3!}+\frac{50}{4!}+\frac{78}{5!}+\cdots$

$=\sum_{n=1}^{\infty}\frac{3n^2+7n+2}{(n+1)!}$

$=\sum_{n=2}^{\infty}\frac{3(n-1)^2+7(n-1)+2}{n!}$

$=\sum_{n=2}^{\infty}\frac{3n^2+n-2}{n!}$

$=3\sum_{n=2}^{\infty}\frac{n^2}{n!}+\sum_{n=2}^{\infty}\frac{n}{n!}-2\sum_{n=2}^{\infty}\frac{1}{n!}$

$=2(2e-1)+(e-1)-2(e-2)=5e$

75 **(a)**

$\sum_{k=1}^{5}\frac{1^3+2^3+\ldots+k^3}{1+3+5+\ldots+(2k-1)}=\sum_{k=1}^{5}\frac{\left(\frac{k(k+1)}{2}\right)^2}{k^2}$

$=\sum_{k=1}^{5}\frac{(k+1)^2}{4}$

$=\frac{2^2+3^2+4^2+5^2+6^2}{4}$

$=\frac{4+9+16+25+36}{4}$

$=\frac{90}{4}=22.5$

77 **(b)**

Given, $b=\frac{a^1}{1}+\frac{a^2}{2}+\frac{a^3}{3}+\ldots\infty$

$\Rightarrow\ b=-\log(1-a)\quad [\because |a|<1]$

$\Rightarrow\ e^{-b}=(1-a)$

$\Rightarrow\ a=\frac{b}{1!}-\frac{b^2}{2!}+\frac{b^3}{3!}-\ldots\infty$

$=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}b^k}{k!}$

78 **(d)**

We have,

$\frac{e^{5x}+e^x}{e^{3x}}=e^{2x}+e^{-2x}=2\left\{1+\frac{2^2x^2}{2!}+\frac{2^4x^4}{4!}\right\}$

This expansion does not contain any odd power of x

$\therefore$ Coefficient of $x^n=0$

79 **(c)**

$\sin^2 B-\sin^2 A=\sin^2 C-\sin^2 B\quad [\because a^2,b^2,c^2$ are in AP]

$\Rightarrow \sin(B+A)\sin(B-A)=\sin(C+B)\sin(C-B)$

$\Rightarrow \sin C(\sin B\cos A-\cos B\sin A)=\sin A(\sin C\cos B-\cos C\sin B)$

$\Rightarrow 2\cot B=\cot A+\cot C$ [divide by $\sin A\sin B\sin C$]

$\Rightarrow \cot A, \cot B, \cot C$ are in AP.

80 **(d)**

Let $S=1+2x+3x^2+4x^3+\ldots\infty$ …(i)

$xS=x+2x^2+3x^3+\ldots\infty$ …(ii)

Subtracting Eq. (ii) from Eq. (i), we get

$(1-x)S = 1 + x + x^2 + x^3 + \ldots \infty$

$\Rightarrow \; S = \frac{1}{(1-x)}\left(\frac{1}{1-x}\right) = \frac{1}{(1-x)^2}$

Integer Answer Type

81 **(12)**

$\sqrt{5} + \sqrt{125} + \sqrt{405} + \sqrt{845} + \cdots = 276\sqrt{5}$

$\Rightarrow \sqrt{5}(1 + \sqrt{25} + \sqrt{81} + \sqrt{169} + \cdots) = 276\sqrt{5}$

$\Rightarrow 1 + 5 + 9 + 13 + \cdots = 276$

$\Rightarrow \frac{n}{2}[2(1) + (n-1)4] = 276$

$\Rightarrow n(4n-2) = 552$

$\Rightarrow 4n^2 - 2n - 552 = 0$

$\Rightarrow 2n^2 - n - 276 = 0$

$\Rightarrow (2n+23)(n-12) = 0$

$\Rightarrow n = -\frac{23}{2}$ or $n = 12$

$\Rightarrow n = 12$ … [n cannot be negative or fraction]

82 **(9)**

Given, $a_1 = 3, m = 5n$ and a_1, a_2 …, is an AP.

$\therefore \quad \frac{S_m}{S_n} = \frac{S_{5n}}{S_n}$ is independent of n.

$= \frac{\frac{5n}{2}[2 \times 3 + (5n-1) - d]}{\frac{n}{2}[2 \times 3 + (n-1)d]}$

$= \frac{5\{(6-d) + 5n\}}{(6-d) + n}$, independent of n

If $\quad 6 - d = 0$

$\Rightarrow \quad d = 6$

$\therefore \quad a^2 = a_1 + d = 3 + 6 = 9$

or If $d = 0$, then $\frac{S_m}{S_n}$ is independent of n.

$\therefore \quad a_2 = 9$

83 **(9)**

Given, $\frac{S_7}{S_{11}} = \frac{6}{11}$ and $130 < t_7 < 140$

$\Rightarrow \quad \frac{\frac{7}{2}[2a + 6d]}{\frac{11}{2}[2a + 10d]} = \frac{6}{11}$

$\Rightarrow \quad \frac{7(2a + 6d)}{(2a + 10d)} = 6$

$\Rightarrow \quad a = 9d$ …(i)

Also, $\quad 130 < t_7 < 140$

$\Rightarrow \quad 130 < a + 6d < 140$

$\Rightarrow \quad 130 < 9d + 6d < 140$ [from Eq. (i)]

$\Rightarrow \quad 130 < 15d < 140$

$\Rightarrow \quad \frac{26}{3} < d < \frac{28}{3}$ [since, d is a natural number]

$\therefore \quad d = 9$

84 **(-8)**

Let

$S_n = \frac{1}{\sqrt{17} - \sqrt{16}} - \frac{1}{\sqrt{18} - \sqrt{17}} + \frac{1}{\sqrt{19} - \sqrt{18}} - \cdots - \frac{1}{\sqrt{144} - \sqrt{143}}.$

By rationalizing the denominator of each term, we get

$\Rightarrow S_n = \frac{\sqrt{17} + \sqrt{16}}{1} - \frac{\sqrt{18} + \sqrt{17}}{1} + \frac{\sqrt{19} + \sqrt{18}}{1} - \cdots - \frac{\sqrt{144} + \sqrt{143}}{1}$

$= \sqrt{17} + \sqrt{16} - \sqrt{18} - \sqrt{17} + \sqrt{19} + \sqrt{18}$

$= \sqrt{16} - \sqrt{144}$

$= 4 - 12 = -8$

85 **(96)**

$\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots = \frac{\pi^4}{90}$

$\Rightarrow \left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\right) + \frac{1}{2^4}\left(\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots\right) = \frac{\pi^4}{90}$

$\Rightarrow \left(\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots\right) + \frac{1}{16}\left(\frac{\pi^4}{90}\right) = \frac{\pi^4}{90}$

$\Rightarrow \frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \cdots = \frac{\pi^4}{90}\left(1 - \frac{1}{16}\right) = \frac{\pi^4}{96}$

$\Rightarrow a = 96$